Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **For examination from 2020**

MARK SCHEME

Maximum Mark: 75

Specimen

This document has **14** pages. Blank pages are indicated.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types.

M	Method mark, given for a valid method applied to the problem. Method marks can still be given even if there are numerical errors, algebraic slips or errors in units. However the method must be applied to the specific problem, e.g. by substituting the relevant quantities into a formula. Correct use of a formula without the formula being quoted earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, given for an accurate answer or accurate intermediate step following a correct method. Accuracy marks cannot be given unless the relevant method mark has also been given.
B	Mark for a correct statement or step.
DM or DB	M marks and B marks are generally independent of each other. The notation DM or DB means a particular M or B mark is dependent on an earlier M or B mark (indicated by *). When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT below).
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures (sf) or would be correct to 3 sf if rounded (1 decimal point (dp) for angles in degrees). As stated above, an A or B mark is not given if a correct numerical answer is obtained from incorrect working.
- Common alternative solutions are shown in the Answer column as: '**EITHER** Solution 1 **OR** Solution 2 **OR** Solution 3 …'. Round brackets appear in the Partial Marks column around the marks for each alternative solution.
- Square brackets [ ] around text show extra information not needed for the mark to be awarded.
- The total number of marks available for each question is shown at the bottom of the Marks column in bold type.

The following abbreviations may be used in a mark scheme.

AG	Answer given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
CAO	Correct answer only (emphasising that no 'follow through' from an error is allowed).
CWO	Correct working only
FT	Follow through after error (see Mark Scheme Notes for further details).
ISW	Ignore subsequent working
OE	Or equivalent form
SC	Special case
SOI	Seen or implied

Question	Answer	Marks	Partial Marks	Guidance
1	Use law of the logarithm of a product, quotient or power	1	**M1**	
	Obtain a correct linear inequality in any form, e.g. $\ln 3 + (3x+1)\ln 2 < \ln 8$	1	**A1**	
	Obtain final answer $x < \dfrac{\ln\frac{4}{3}}{\ln 8}$, or equivalent	1	**A1**	Accept, for example, $\dfrac{2}{3} - \dfrac{\ln 3}{3\ln 2}$
		3		

Question	Answer	Marks	Partial Marks	Guidance
2(a)	State a correct unsimplified version of the x or x^2 term	1	**M1**	Symbolic coefficients, e.g. $\begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$, are not sufficient for the M mark.
	State correct first two terms $1 - x$	1	**A1**	
	State the next term $+ 2x^2$	1	**A1**	
		3		
2(b)	$\lvert x \rvert < \frac{1}{3}$	**1**	**B1**	OE

Question	Answer	Marks	Partial Marks	Guidance
3(a)	Make a recognisable sketch graph of $y = \lvert 2x - 3 \rvert$	**1**	**B1**	
3(b)	**EITHER** Solution 1 Find x-coordinate of intersection with $y = 3x - 1$	1	**(M1**	
	Obtain $x = \frac{4}{5}$	1	**A1**	
	State final answer $x > \frac{4}{5}$ only	1	**A1)**	
	OR Solution 2 Solve the linear inequality $3x - 1 > 3 - 2x$, or corresponding equation	1	**(M1**	
	Obtain critical value $x = \frac{4}{5}$	1	**A1**	
	State final answer $x > \frac{4}{5}$ only	1	**A1)**	
	OR Solution 3 Solve the quadratic inequality $(3x - 1)^2 > (3 - 2x)^2$, or corresponding equation	1	**(M1**	
	Obtain critical value $x = \frac{4}{5}$	1	**A1**	Unsupported answer receives 0 marks
	State final answer $x > \frac{4}{5}$ only	1	**A1)**	
	Available marks	**3**		

Question	Answer	Marks	Partial Marks	Guidance
4(a)	State or imply $\frac{dx}{dt} = 2e^{2t-3}$ or $\frac{dy}{dt} = \frac{4}{t}$	1	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	1	**M1**	
	Obtain $\frac{dy}{dx} = \frac{4}{2te^{2t-3}}$, or equivalent	1	**A1**	
	Set $t = a$, equate gradient to 2 and obtain the given answer	1	**A1**	AG
		4		
4(b)	Calculate $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	1	**M1**	
	Complete the argument by considering the signs of the correct calculated values	1	**A1**	
		2		
4(c)	Use the iterative formula correctly at least once	1	**M1**	
	Obtain final answer 1.35	1	**A1**	
	Show sufficient iterations to 4 dp to justify 1.35 to 2 dp, **OR** show there is a sign change in the interval (1.345, 1.355)	1	**A1**	
		3		

Question	Answer	Marks	Partial Marks	Guidance
5(a)	State correct derivative $1-\dfrac{1}{1+x^2}$	1	**B1**	
	Rearrange and obtain the given answer	1	**B1**	AG
		2		
5(b)	Integrate by parts and reach $ax^2\tan^{-1}x+b\displaystyle\int\frac{x^2}{1+x^2}\,\mathrm{d}x$	1	**M1**	
	Obtain $\frac{1}{2}x^2\tan^{-1}x-\frac{1}{2}\displaystyle\int\frac{x^2}{1+x^2}\,\mathrm{d}x$, or equivalent	1	**A1**	
	Obtain complete indefinite integral $\frac{1}{2}(x^2\tan^{-1}x-x+\tan^{-1}x)$, or equivalent	1	**A1**	
	Substitute limits having integrated twice	1	**M1**	
	Obtain the given answer correctly	1	**A1**	AG
		5		

Question	Answer	Marks	Partial Marks	Guidance
6(a)	**EITHER** Solution 1 Multiply the numerator and denominator of $\frac{u}{v}$ by $4 - 2\text{i}$, or equivalent	1	**(M1**	
	Simplify the numerator to $10 + 10\text{i}$ or the denominator to 20	1	**A1)**	
	OR Solution 2 Obtain two equations in x and y, and solve for x or for y	1	**(M1**	
	Obtain $x = \frac{1}{2}$ or $y = \frac{1}{2}$, or equivalent	1	**A1)**	
	Obtain final answer $\frac{1}{2} + \frac{1}{2}\text{i}$	1	**A1**	Unsupported answer receives 0 marks
	Available marks	**3**		
6(b)	State argument is $\frac{1}{4}\pi$ (or 0.785 radians or 45°)	**1**	**B1FT**	
6(c)	State that OC and BA are equal (in length)	1	**B1**	
	State that OC and BA are parallel or have the same direction	1	**B1**	
		2		
6(d)	**EITHER** Solution 1 Use angle $AOB = \arg u - \arg v = \arg\left(\frac{u}{v}\right)$	1	**(M1**	
	Obtain given answer (or 45°)	1	**A1)**	AG
	OR Solution 2 Obtain $\tan AOB$ from gradients of OA and OB and $\tan(A \pm B)$ formula	1	**(M1**	
	Obtain given answer (or 45°)	1	**A1)**	AG
	Available marks	**2**		

Question	Answer	Marks	Partial Marks	Guidance
7(a)	Use $\cos(A + B)$ formula to obtain an expression in terms of $\cos x$ and $\sin x$	1	**M1**	
	Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent	1	**A1**	
	Obtain $R = 2.236$	1	**A1**	
	Use trig formula to find α	1	**M1**	
	Obtain $\alpha = 71.57°$ with no errors seen	1	**A1**	
		5		
7(b)	Evaluate $\cos^{-1}\left(\frac{2}{2.236}\right)$ to at least 1 dp	1	**B1FT**	
	Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$	1	**M1**	
	Obtain answer, e.g. $x = 315°$	1	**A1**	
	Obtain second answer, e.g. $261.9°$ and no others in the given interval	1	**A1**	
		4		

Question	Answer	Marks	Partial Marks	Guidance
8(a)	State $\overrightarrow{ON} = 2\mathbf{j} + \mathbf{k}$	1	**B1**	
	Use $\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC}$	1	**M1**	
	Obtain $\overrightarrow{CM} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	1	**A1**	
		3		
8(b)	Carry out correct process for evaluating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{CM}$	1	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	1	**M1**	
	Obtain answer 72.7° or 1.27 radians	1	**A1**	
		3		

Question	Answer	Marks	Partial Marks	Guidance
8(c)	**EITHER** Solution 1 Taking general point P of ON to have position vector $\lambda(2\mathbf{j}+\mathbf{k})$, form an equation in λ by ***either*** equating the scalar product of $\overrightarrow{MP}$ and $\overrightarrow{ON}$ to zero, ***or*** applying Pythagoras in triangle OPM, ***or*** setting the derivative of $\lvert\overrightarrow{MP}\rvert$ or $\lvert\overrightarrow{MP}\rvert^2$ to zero	1	**(M1**	
	Solve and obtain $\lambda = \frac{4}{5}$	1	**A1**	
	Substitute for λ and calculate MP	1	**M1**	
	Obtain the given answer	1	**A1)**	AG
	OR Solution 2 Use $\dfrac{\overrightarrow{OM}.\overrightarrow{ON}}{\lvert\overrightarrow{ON}\rvert}$ to find projection OQ of OM on ON	1	**(M1**	
	Obtain $OQ = \dfrac{4}{\sqrt{5}}$	1	**A1**	
	Use Pythagoras in triangle OMQ to find MQ	1	**M1**	
	Obtain the given answer	1	**A1)**	AG
	OR Solution 3 Using a relevant scalar product, find the cosine of angle MON or angle ONM	1	**(M1**	
	Obtain $\cos MON = \frac{4}{5}$ or $\cos ONM = \frac{3}{5}$	1	**A1**	
	Use trig to find the length of the perpendicular	1	**M1**	
	Obtain the given answer	1	**A1)**	
	Available marks	**4**		AG

Question	Answer	Marks	Partial Marks	Guidance
9(a)	Use product rule	1	**M1**	
	Obtain correct derivative in any form, e.g. $4\sin 2x\cos 2x\cos x - \sin^2 2x\sin x$	1	**A1**	
	Equate derivative to zero and use a double angle formula	1	**M1***	
	Reduce equation to one in a single trig function	1	**DM1**	
	Obtain a correct equation in any form, e.g. $10\cos^3 x = 6\cos x$, $4 = 6\tan^2 x$, or $4 = 10\sin^2 x$	1	**A1**	
	Solve and obtain $x = 0.685$	1	**A1**	Unsupported answer receives 0 marks
		6		
9(b)	Using $\mathrm{d}u = \pm\cos x\,\mathrm{d}x$, or equivalent, express integral in terms of u and $\mathrm{d}u$	1	**M1**	
	Obtain $\int 4u^2(1-u^2)\,\mathrm{d}u$	1	**A1**	
	Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	1	**M1**	
	Obtain answer $\frac{8}{15}$ (or 0.533)	1	**A1**	Unsupported answer receives 0 marks
		4		

Question	Answer	Marks	Partial Marks	Guidance
10(a)	State or imply $\frac{dx}{dt} = k(10 - x)(20 - x)$ and show $k = 0.01$	**1**	**B1**	
10(b)	Separate variables and attempt integration of at least one side	1	**M1**	
	Carry out an attempt to find A and B such that $\frac{1}{(10-x)(20-x)} \equiv \frac{A}{10-x} + \frac{B}{20-x}$	1	**M1**	
	Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent	1	**A1**	
	Integrate and obtain $-\frac{1}{10}\ln(10 - x) + \frac{1}{10}\ln(20 - x)$, or equivalent	1	**A1FT**	
	Integrate and obtain term $0.01t$, or equivalent	1	**A1**	
	Evaluate a constant, or use limits $t = 0$, $x = 0$ in a solution containing terms of the form $a\ln(10 - x)$, $b\ln(20 - x)$ and ct	1	**M1**	
	Obtain answer in any form, e.g. $-\frac{1}{10}\ln(10 - x) + \frac{1}{10}\ln(20 - x) = 0.01t + \frac{1}{10}\ln 2$	1	**A1FT**	
	Use laws of logarithms correctly to remove logarithms	1	**M1**	
	Rearrange and obtain $x = \frac{20(e^{0.1t} - 1)}{2e^{0.1t} - 1}$, or equivalent	1	**A1**	
		9		
10(c)	State that x approaches 10	**1**	**B1**	

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